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Physics

By

ERICH HAUSMANN, E.E., Sc.D.
THOMAS POTTS PROFESSOR OF PHYSICS AND DEAN OF GRADUATE STUDY
POLYTECHNIC INSTITUTE OF BROOKLYN

AND

EDGAR P. SLACK, S.B., M.S.
ASSOCIATE PROFESSOR OF PHYSICS
POLYTECHNIC INSTITUTE OF BROOKLYN

SECOND EDITION—FOURTEENTH PRINTING

NEW YORK
D. VAN NOSTRAND COMPANY, INC.
250 FOURTH AVENUE

The packing case could have been moved much more easily by rolling it along on two pieces of pipe, replacing the sliding friction by rolling friction, which has a much lower value. Thus, the maximum force of friction might be reduced to perhaps 30 lb., including the effects at both top and bottom of the rollers. Under these circumstances a horizontal push of 30 lb. would balance friction, and any harder push would start the case moving. Rolling friction is the opposition which occurs when one body rolls upon another, and is probably due to slight deformations of the bodies at the point of contact; it is treated in the same way as sliding friction.

45. Coefficient of Friction. — The maximum force of friction which one body is capable of exerting upon another is directly proportional to the normal or perpendicular force with which their surfaces are pressed together. Thus, if f is the maximum force of friction between two bodies which are pressed together with a force N perpendicular to their contacting surfaces, then $f \propto N$, or

$$f = \mu N$$

where the proportionality factor μ (mu) is a quantity which is called the *coefficient of friction* between the surfaces. Transposing, this expression becomes

$$\mu = \frac{f}{N} \qquad (34)$$

from which the coefficient of friction is defined as *the ratio of the maximum force of friction between two bodies to the normal force pressing the surfaces together.* It is evident from equation (34), that since f and N are both forces, they may be expressed in any desired force unit, so long as the same unit is used for both, and that μ is a pure numeric, having no unit. Since the frictional force f is always in the plane of sliding, and the normal force N pressing the two surfaces into contact is perpendicular to this plane, it follows that f and N are always at right angles to each other.

The coefficient of sliding friction may be taken as a constant for any two surfaces, depending only on the materials involved and the condition of the surfaces in contact. Some representative values for dry surfaces appear in the accompanying table.

Coefficients of Sliding Friction

Wood on wood	0.25 to 0.5
Metals on wood	0.2 to 0.6
Metals on metals	0.15 to 0.2
Leather on oak	0.27 to 0.38
Leather on metals	0.56

For surfaces which are carefully machined and thoroughly lubricated, the coefficient of sliding friction is much smaller, 0.005 being a representative value.

As a typical problem involving sliding friction, consider a sled being drawn on level snow by a constant force of 10 lb. applied at an angle 25° upward from the horizontal. If the sled with its load weighs 60 lb. and if the coefficient of friction between the sled and the snow is 0.05, find the time required to travel 100 ft., starting from rest.

Following the procedure suggested in § 43, the sled, shown in part I of Fig. 50, is chosen as the body to be considered, and is represented at O

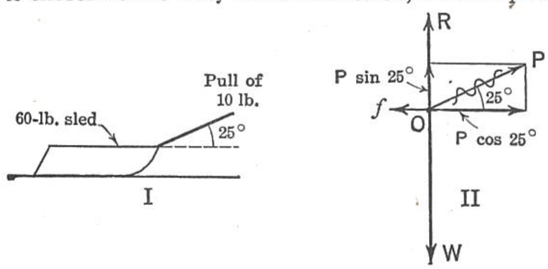

FIG. 50. Sled drawn along horizontal surface

in the force diagram forming part II of the figure. The forces acting on the sled are as follows: the downward pull of gravity, W (60 lb.); the pull P (10 lb.) applied in the direction shown; the upward reaction of the snow, R (unknown); and the backward force of friction, f (also unknown). The pull P is resolved into a horizontal component P cos 25° = 10 × 0.906 = 9.06 lb., which shows how much of the pull is effective in drawing the sled horizontally, and a vertical component P sin 25° = 10 × 0.422 = 4.22 lb., which shows how much the sled is being pulled up, reducing its pressure on the snow. The pull P may now be disregarded and may be crossed out, since it is replaced by its components. The net force which the sled exerts downward on the snow is 60 − 4.22 = 55.78 lb., and hence the upward thrust R of the snow on the sled is also 55.78 lb., this being the normal force between the sliding surfaces. From equation (34), $f = \mu N = 0.05 \times 55.78 = 2.79$ lb. It is noted that the upward forces 55.78 lb. and 4.22 lb. exactly balance the downward force of 60 lb., and thus the unbalanced force acting on the sled is toward the right and amounts to 9.06 − 2.79 or 6.27 lb. The acceleration of the sled, from equation (33), is consequently

$$a = \frac{F \times g}{W} = \frac{6.27 \text{ lb.} \times 32 \dfrac{\text{ft.}}{\text{sec.}^2}}{60 \text{ lb.}} = 3.35 \frac{\text{ft.}}{\text{sec.}^2}.$$

Substituting this acceleration value in equation (21)

$$s = v_0 t + \tfrac{1}{2} a t^2 = 0 + \tfrac{1}{2} a t^2$$

the time to travel 100 ft. is found to be

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 100 \text{ ft.}}{3.35 \dfrac{\text{ft.}}{\text{sec.}^2}}} = \sqrt{59.8 \text{ sec.}^2} = 7.73 \text{ sec.}$$

The coefficients of rolling friction are much smaller than for sliding friction; some typical values appear in the accompanying table:

Coefficients of Rolling Friction

Cast-iron wheels on rails...........	0.004
Ball bearings in rolling contact......	0.001 to 0.003
Roller bearings in rolling contact....	0.002 to 0.007

It is because rolling friction is so small compared with sliding friction that wheels are used instead of runners on wagons, that gear teeth are designed so as to roll together when meshed, and that for many purposes ball bearings are preferred to those of the sliding, or sleeve, type. In the sleeve bearing the shaft slides, while in the ball bearing it rolls, and although the balls slide somewhat upon each other, the force of friction is much reduced.

In addition to sliding and rolling friction, retardation is also offered to the motion of an object by the air, and this is of particular importance in the propulsion of ships, automobiles, trains, and aircraft. This retardation increases with velocity, and at relatively low speeds is often regarded as being proportional to the velocity. Taking v as the velocity of an object, the retarding force would be expressed as $f \propto v$ or $f = Rv$, where R is the proportionality factor which may be termed mechanical resistance. If this frictional force is included in equation (31) for accelerated motion, the total force required to produce an acceleration $a = \frac{dv}{dt}$ [§ 28] in a mass m is

$$P = ma + Rv = m\frac{dv}{dt} + Rv$$

In starting, $v = 0$ and the entire unbalanced force produces acceleration; when the body has accelerated to the maximum speed permitted by the frictional drag, $v =$ constant, $\frac{dv}{dt} = 0$, and the entire force is exerted in maintaining the speed acquired against the mechanical resistance. The foregoing equation is useful in studying motion at relatively low speeds; the behavior at high speeds is complicated by the fact that air resistance is then more nearly proportional to the square of the speed.

For further treatment of static, sliding, and rolling friction, and of air resistance, the student is referred to advanced texts.

CIRCULAR MOTION

46. Force Involved in Circular Motion. — In accordance with Newton's First Law of Motion, a moving body left to itself will travel in a straight line; a body will not move around a curve unless a lateral force is exerted upon it. When a locomotive encounters a curve, its forward motion causes the flanges on the wheels to press outwardly against the edge of the outer rail, and consequently the rail presses inwardly against the flanges; the locomotive, under the action of this inward force, undergoes a change of direction and continues to follow the track.

There are many other examples that show the existence of this lateral force which is necessary for motion along a curve. If a stone is whirled around at the end of a cord, it pulls outwardly on the cord, whereupon the cord becomes taut and pulls inwardly upon the stone. In the same way, the earth, in moving along its orbit, is always being drawn inward by the gravitational attraction of the sun. A little thought will show why an automobile sometimes skids when the driver tries to make a sharp turn on a slippery street.

The motion of a body traveling around a circular curve with constant speed is of special interest, for *in such circular motion the moving object acts upon the restraining agent with a constant force directed radially outward from the center; this outward push or pull is called the centrifugal force.* Since for any action there is always an equal and opposite reaction, *the restraining agent exerts an equal inward force upon the moving object, and this is called the centripetal force.* In all motion on curves, the centrifugal and centripetal forces are equal and opposite, and both are exerted in the plane of rotation. *Although equal, these forces cannot balance each other, because they are not exerted upon the same object.*

An unbalanced force always produces acceleration, and thus the centripetal force acting upon a body in circular motion continually accelerates it toward the center of the circle; in fact, it is